Info Letter No. 5

Zero sequence voltage in three-phase networks

1. Zero sequence voltage in three-phase networks

With balanced network operation and inequality of the impedances in the consumer circuit, the phase voltages of the two circuits, and thus the neutral points are no longer congruent.

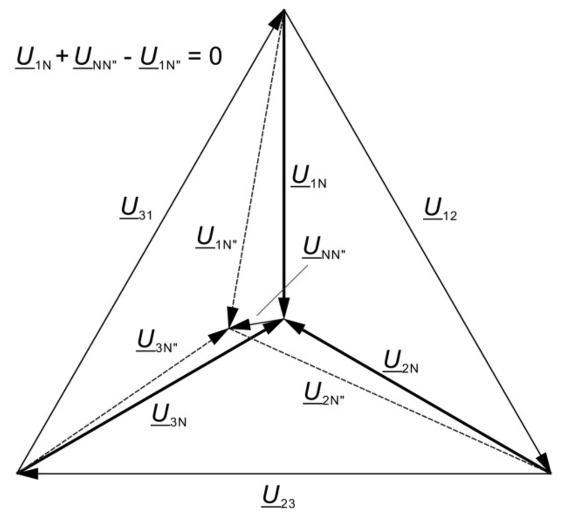

Figure 1
Voltage indicator in a three-phase three-wire network. Between the neutral points, there is a voltage difference, which is referred to as the zero sequence voltage and the amount depends on the inequality of the impedances in the consumer circuit.

1.1 Three-phase three-wire network

The geometric sum of the complex effective values of the phase currents (see Figure 2) is zero and thus:

$$\frac{\underline{U}_{1N} + \underline{U}_{NN"}}{\underline{Z}"_1} + \frac{\underline{U}_{2N} + \underline{U}_{NN"}}{\underline{Z}"_2} + \frac{\underline{U}_{3N} + \underline{U}_{NN"}}{\underline{Z}"_3} = 0 \quad (1)$$

As a result:

$$\underline{U}_{NN"} = \frac{1}{3}\left[\left(\underline{U}_{1N"} + \underline{U}_{2N"} + \underline{U}_{3N"}\right) - \left(\underline{U}_{1N} + \underline{U}_{2N} + \underline{U}_{3N}\right)\right] \quad (2)$$

With symmetrical network operation $\underline{U}_{1N} + \underline{U}_{2N} + \underline{U}_{3N} = 0$ and thus the voltage $\underline{U}_{NN“}$ equal to the zero component $\underline{U}_0$, which is therefore also called the zero sequence voltage.

$$\underline{U}_{NN"} = \frac{1}{3}\left(\underline{U}_{1N"} + \underline{U}_{2N"} + \underline{U}_{3N"}\right) \quad (3)$$

Then also:

$$\underline{U}_{NN"} = -\frac{1}{3}\left[\underline{I}_1\left(\underline{Z}"_2 - \underline{Z}"_1\right) + \underline{I}_3\left(\underline{Z}"_2 - \underline{Z}"_3\right)\right] \quad (4)$$

The effect on the zero sequence voltage of unequal impedances in the consumer circuit (which in turn cause asymmetric currents) is directly recognizable from this equation.

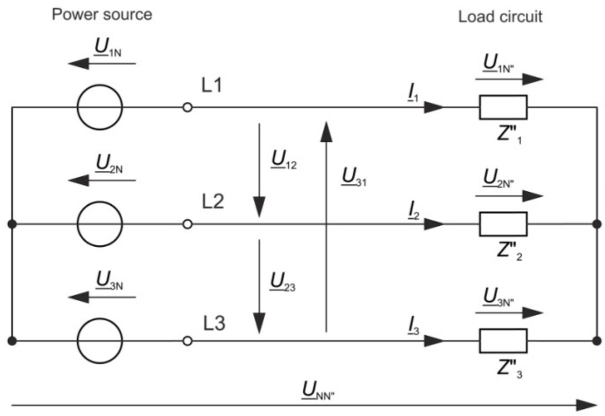

Figure 2
Voltages in three-phase three-wire systems (Short-circuit impedances of the power source and the serial impedance of the cable are ignored)

Example 1

Three-phase three-wire network; sinusoidal alternating values; voltages at the measurement point / measurement equipment:

$\underline{U}_{12}$ = 20287 e $^{j30°}$ V;

$\underline{U}_{23}$ = 20162 e $^{-j90°}$ V;

$\underline{U}_{31}$ = 20345 e $^{j150°}$ V;

Consumer circuit:
$\underline{Z}“_1$ = 30 e $^{j25°}\Omega$; $\underline{Z}“_2$ = 28 e $^{j30°}\Omega$; $\underline{Z}“_3$ = 32 e $^{j35°}\Omega$

(Star connection; asymmetry 7 %);

What is the value of the zero sequence voltage?

Result (calculation with program E-1.4.1)

Currents:
$\underline{I}_{1“}$ = 384 e $^{-j25°}$A; $\underline{I}_{2“}$ = 422 e $^{-j151°}$A; $\underline{I}_{3“}$ = 368 e $^{j87°}$A

Voltages in the consumer circuit:

$\underline{U}_{1N“}$ = 11508 e $^{-j0°}$ V;

$\underline{U}_{2N“}$ = 11815 e $^{-j121°}$ V;

$\underline{U}_{3N“}$ = 11782 e $^{j122°}$ V

Zero sequence voltage:
$\underline{U}_{N'N“}$ = 254 e $^{-j171°}$ V (2 % of $\underline{U}_{1N“}$; corresponding to the voltage difference between the neutral point of the consumer circuit and the neutral point of the power source).

1.2 Three-phase four-wire network

Just as in the three-phase three-wire system, in the three-phase four-wire system the following also applies for symmetrical network operation:

$$\underline{U}_{\mathrm{NN"}} = \frac{1}{3}\left(\underline{U}_{\mathrm{1N"}} + \underline{U}_{\mathrm{2N"}} + \underline{U}_{\mathrm{3N"}}\right) = \underline{I}_{\mathrm{NN"}}\ \underline{Z}"_{\mathrm{N}} \qquad (5)$$

$\underline{U}_{NN''} = \underline{U}_0$ (zero component)

If instead of the impedances of the consumer circuit, the corresponding admittances are used, the zero sequence voltage is given by:

$$\underline{U}_{\mathrm{NN"}} = -\frac{\underline{U}_{\mathrm{1N}}\ \underline{Y}"_1 + \underline{U}_{\mathrm{2N}}\ \underline{Y}"_2 + \underline{U}_{\mathrm{3N}}\ \underline{Y}"_3}{\underline{Y}"_1 + \underline{Y}"_2 + \underline{Y}"_3 + \underline{Y}"_{\mathrm{N}}} \qquad (6)$$

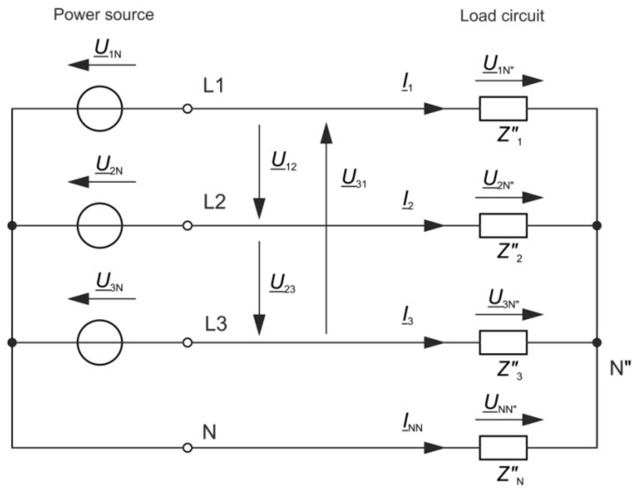

Figure 3
Voltages in three-phase four-wire systems
(Short-circuit impedances of the power source and the serial impedance of the cable are ignored)

Example 2

Three-phase four-wire network; sinusoidal alternating values; voltages at the measurement point / measurement equipment:

U_{12} = 401.2 V; U_{23} = 402.5 V; U_{31} = 401.9 V;
(30° is chosen for the angle of U_{12}).

Consumer circuit:

$\underline{Z}''_1 = 17\ e^{j28°}\ \Omega$;

$\underline{Z}''_2 = 23\ e^{j30°}\ \Omega$;

$\underline{Z}''_3 = 28\ e^{j34°}\ \Omega$;

$\underline{Z}''_N = 1{,}0\ e^{j4°}\ \Omega$ (Star connection; asymmetry 25 %).

What is the value of the zero sequence voltage?

Result (calculation with program E-1.4.2)

Voltages in the consumer circuit:

$\underline{U}_{1N''} = 226.8\ e^{j0{,}6°}$ V;

$\underline{U}_{2N''} = 232.4\ e^{-j121{,}2°}$ V;

$\underline{U}_{3N''} = 236.9\ e^{j120{,}7°}$ V;

Zero sequence voltage:
$\underline{U}_{NN''} = \underline{U}_0 = 5\ e^{-j152°}$ V (2 % of $\underline{U}_{1N''}$)

1.3 Measurement of the zero sequence voltage

The zero sequence voltage can be determined either from the phase-earth voltages measured at any point in the network covered by three voltage transformers (sum of the instantaneous values, null component of the voltages) or directly at the auxiliary winding of an earth-fault compensation coil.

For the measurement of the zero sequence voltage, single pole insulated voltage transformers are equipped with an additional auxiliary winding (named da - dn, earlier e - n). Between the beginning and the end (the open ends) of the auxiliary windings connected in series, the total value of the three phase-to-ground voltages can be measured. The sizing of the auxiliary windings is usually such that a with a saturated single-pole earth fault the effective values of the zero sequence voltage and phase-phase voltage are equal.

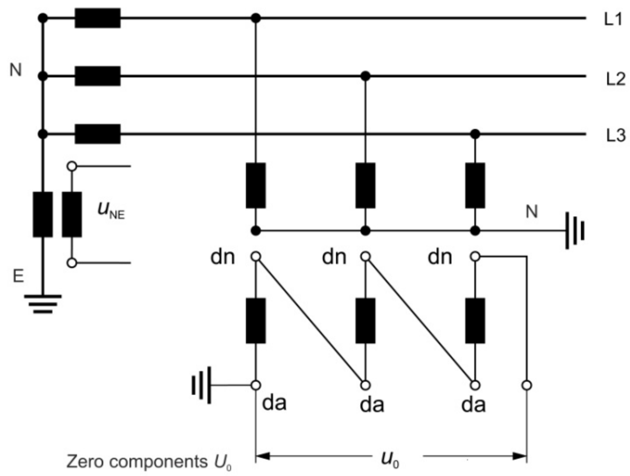

Figure 4
Measurement transformer circuitry for measuring the zero sequence voltage; at the series circuit of the auxiliary windings the terminal da (e) of the first winding is preferably earthed.

Note

When there is an earth fault, the phase-earth voltages of the earth-fault free conductor increase by a factor √3, so that the total voltage on the secondary side of the voltage transformer with the same dimensions of all the windings is 100 V·√3.

However, to obtain the value of the total voltage of 100 V with an earth fault, the voltage of the auxiliary winding

$$\frac{100\mathrm{V}}{\sqrt{3}\cdot\sqrt{3}} = \frac{100}{3}$$ *must be*

(Notation on the type plate of the voltage transformer).

Effect of the measurement transformer error

Because of the amplitude and angular error of current transformer, the sum of the three voltages also differs marginally from zero in the earth fault-free three-phase three-wire network.

Example 3

Three-phase three-wire network; sinusoidal alternating values and the secondary windings of 3 voltage transformers are connected in series.

$\underline{U}_{1E}$ = 100 V, Error: + 0.4 % / – 0.3°;

$\underline{U}_{2E}$ = 100 V, Error: – 0.25 % / – 0.4°;

$\underline{U}_{3E}$ = 100 V, Error: + 0.3 % / + 0.3°

How big is the variance of the total voltage from zero?

Result (calculation with program E-1.7.1)
Instead of $\underline{U}_{\Sigma}$ = 0 and φ_{Σ} = 0°, the following is displayed:
$\underline{U}_{\Sigma}$* = 0.68 V. This corresponds to 0.7 % of 100 V.

φ^*_{Σ} = 177°

1.4 Zero sequence voltage with an earth fault

A high zero sequence voltage is an indication of an earth fault in the network, because in normal network conditions, the asymmetry of the phase-earth impedance and thus the zero sequence voltage is low, in earth-fault free networks it is only a few percent of the phase-earth voltage. Because of the significantly improved symmetry of the phase-earth impedance of cables in these networks the zero sequence voltage is even lower than in overhead transmission networks. - With a saturated earth fault the zero sequence voltage can reach the full value of the phase-neutral point voltage. Due to the earth fault of a conductor, the corresponding phase-earth impedance is largely short-circuited in the entire, electrically connected network. The increase in the zero sequence voltage thus occurs independently of the location of the earth fault. The value and angle are however location-dependent; they are determined by the phase-phase voltages and the phase-earth impedances at the measurement point. The waveform and frequency of the network voltage and zero sequence voltage are the same after the end of the short transient process caused by the earth fault.

In the three-phase network with earth fault compensation:

$$\underline{U}_{\mathrm{NE}} = \frac{1}{3}\left[\left(\underline{U}_{1\mathrm{E}} + \underline{U}_{2\mathrm{E}} + \underline{U}_{3\mathrm{E}}\right) - \left(\underline{U}_{1\mathrm{N}} + \underline{U}_{2\mathrm{N}} + \underline{U}_{3\mathrm{N}}\right)\right] \qquad (7)$$

With balanced network operation $\underline{U}_{1N}$ + $\underline{U}_{2N}$ + $\underline{U}_{3N}$ = 0 and thus the voltage $\underline{U}_{NE}$ = $\underline{U}_0$ (null component)

$$\underline{U}_0 = \frac{1}{3}\left(\underline{U}_{1\mathrm{E}} + \underline{U}_{2\mathrm{E}} + \underline{U}_{3\mathrm{E}}\right) \qquad (8)$$

The zero sequence voltage also arises from the apparent values between phase and earth, the apparent value of the earth fault fire compensation coil and from the phase-nuetral point voltages.

When there is an earth fault of a conductor, the impedance at the earth fault location acts in parallel to the phase-earth impedance of that conductor.

$$\underline{U}_{\mathrm{NE}} = -\frac{\underline{U}_{1\mathrm{N}}\,\underline{Y}_{1\mathrm{E}} + \underline{U}_{2\mathrm{N}}\,\underline{Y}_{2\mathrm{E}} + \underline{U}_{3\mathrm{N}}\,\underline{Y}_{3\mathrm{E}}}{\underline{Y}_{1\mathrm{E}} + \underline{Y}_{2\mathrm{E}} + \underline{Y}_{3\mathrm{E}} + \underline{Y}_{\mathrm{NE}}} \qquad (9)$$

$\underline{Y}_{iE}$ apparent value between conductor Li and earth; with an earth fault: $\underline{Y}_{iE}$ // $\underline{Y}_{iF}$

$\underline{Y}_{NE}$ apparent value between the neutral point of the transformer and earth (earth fault compensation coil)

With symmetry of the voltages $\underline{U}_{iN}$, both with the same values $\underline{Y}_{iE}$ and earth fault-free network status, the numerator of the fraction and thus the zero sequence voltage occurring at the earth compensation coil becomes zero.

With a saturated earth fault in one conductor, the associated value $\underline{Y}_{iE}$ acquires very high values, so that $\underline{U}_{NE} \approx \underline{U}_{iN}$ (for $\underline{Z}_{iE}$ = 0, thus $\underline{Y}_{iE} = \infty$ there is an *indefinite term*!).

Example 4

Three-phase three-wire network; sinusoidal alternating values; earth fault compensation; earth fault on conductor L1.

Voltages at the measurement location / measurement equipment:
$\underline{U}_{12}$ = 20.16 e $^{j31°}$ kV;

$\underline{U}_{23}$ = 20.01 e $^{-j90°}$kV;

$\underline{U}_{31}$ = 20.24 e $^{j151°}$kV;

Phase-earth capacitances:
C_1 = 6.2 µF; C_2 = 5.9 µF; C_3 = 6.1 µF
(asymmetry 3 %);

Conductor derivations (conductor-earth):
R_1 = 102 kΩ; R_2 = 105 kΩ; R_3 = 98 kΩ
(asymmetry 4 %);

Earth fault on conductor L1: Resistance Z_F = 25 Ω;

Earth fault compensation coil:
Impedance $\underline{Z}_{NE}$ = 173 e $^{j86°}$ Ω (Inductance L = 0.55 H; Copper resistance R = 9 Ω; Iron losses R = 7000 Ω).

What are the value and angle of the phase-earth voltages and zero sequence voltage?

Result (calculation with program E-1.7.2)
$\underline{U}_{1E}$ = 0.15 e $^{-j13^\circ}$ kV;

$\underline{U}_{2E}$ = 20.05 e $^{-j150^\circ}$ kV;

$\underline{U}_{3E}$ = 20.09 e $^{j150^\circ}$ kV

Zero sequence voltage $\underline{U}_{NE}$ = 11.55 e $^{-j180^\circ}$ kV

The phase-phase voltages are not changed by the earth fault, therefore:
$\underline{U}_{12} = \underline{U}_{1E} - \underline{U}_{2E}$
= 0.15 e $^{-j13^\circ}$ kV – 20.05 e $^{-j150^\circ}$ kV = 20.16 e $^{j31^\circ}$ kV

$\underline{U}_{23} = \underline{U}_{2E} - \underline{U}_{3E}$
= 20.05 e $^{-j150^\circ}$ kV – 20.09 e $^{j150^\circ}$ kV = 20.01 e $^{-j90^\circ}$ kV

$\underline{U}_{31} = \underline{U}_{3E} - \underline{U}_{1E}$
= 20.09 e $^{j150^\circ}$ kV – 0.15 e $^{-j13^\circ}$ kV = 20.24 e $^{j150^\circ}$ kV

1.5 Voltage gradient in the event of an earth fault

In the event of an earth fault, three tasks run at the same time (see fig. 5) run within a very short period of time.

- The phase-earth-voltage of the conductor with an earth fault breaks down; with a saturated earth fault this voltage drops to zero.
- The instantaneous values of the phase-earth voltages of the two earth fault-free conductors jump to the instantaneous value of the associated phase-phase voltage.
- The instantaneous value of the zero sequence voltage suddenly rises from the normal operating value (several volts) to a value that, in a saturated earth fault, corresponds to the instantaneous value of the phase-neutral point voltage.

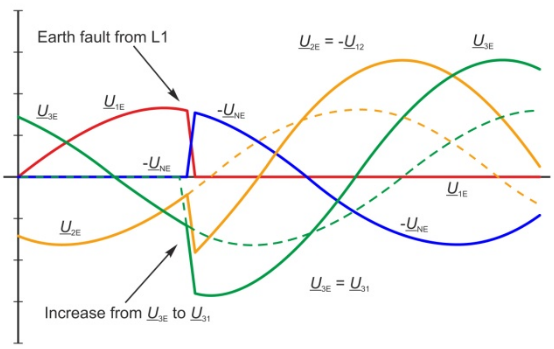

Figure 5
Change in the instantaneous values u_{1N}, u_{2N}, u_{3N}, u_{NE} before and after a single-pole earth fault

This "shifting" occurring during an earth fault causes a short-term high-frequency vibration (transient, ignition oscillation), which is added to the fundamental frequency of the zero sequence voltage, thus significantly determining their variation over time. For the consideration of the stationary state (continuous earth fault) this settling process has no significance.

After the occurrence of the earth fault, the zero sequence voltage changes with a reversed sign in direct continuation of the previous course (amplitude and angle) of the phase-earth voltage that has now decreased to zero.

Phase-phase voltages
In networks with fault compensation a conductor of the associated line from the energy source is not short-circuited by an earth fault (as opposed to the rigid earthing of the neutral point of the energy source). Such line voltages and thus also the phase-phase voltages remain unchanged with a single-pole continuous earth fault.

If there is a continuous saturated earth fault on, for example, conductor L1 (high inequality of the phase-earth impedance), then with symmetric phase-phase voltages:

$$\underline{U}_{1E} = \underline{U}_{1N} + \underline{U}_{NE} = 0$$

$$\underline{U}_{2E} = \underline{U}_{1E} - \underline{U}_{12} = -\underline{U}_{12} = \underline{U}_{12}\ e^{-j150^\circ}$$

$$\underline{U}_{3E} = \underline{U}_{2E} - \underline{U}_{23} = -\underline{U}_{12} - \underline{U}_{23} = \underline{U}_{31}\ e^{j150^\circ}$$

i.e., the phase-earth voltages of both earth fault-free conductors have the same values as the associated phase-phase voltages; the angles of $\underline{U}_{3E}$ and $\underline{U}_{31}$ are the same, those of $\underline{U}_{2E}$ and $\underline{U}_{12}$ have almost the same value, but inverse signs (see Figure 5).

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The Excel programs used for the examples can be obtained from:
www.a-eberle.de
(Download Center)

The series will be continued.
We will gladly supply missing Info Letters at any time!

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