Info Letter No. 9

Voltage regulation with step transformers in parallel to busbars (Part 1)

Within a station, the transformers are in close proximity. Therefore, the line impedances between the secondary terminals of the parallel operating transformers are initially ignored.

General dependencies in parallel operation

To vividly portray the generally dependencies and limits to be observed on the parallel operation of transformers, it is sufficient to clarify these relationships in two parallel operated power sources.

To increase the available power by parallel connection of voltage sources, the total output is ideally equal to the arithmetic sum of the individual outputs. To achieve the maximum total output, the individual voltage sources must however have the same data. With unequal no-load voltages within the parallel circuit, a compensating current flows that changes the proportions between the load current and nominal output. These restrictions reduce the maximum attainable total output and apply, in principle, for the parallel operation of DC and AC voltage sources (batteries, generators, transformers).

The main tasks for parallel operation are therefore:

- Avoidance/minimization of the circulating current
- Avoid unequal relative loading of the voltage sources in parallel operation of direct voltage sources

Parallel operation of DC voltage sources

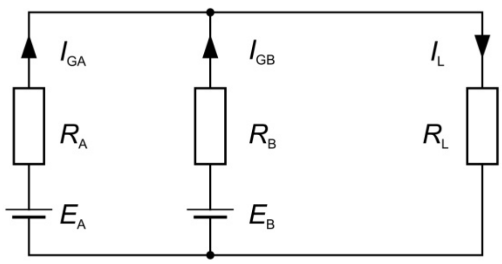

Figure 1 Parallel circuit of two DC voltage sources

Voltage source A:
EMF E_A, internal resistance R_A, current I_{GA}

Voltage source B:
EMF E_B, internal resistance R_B, current I_{GB}

Current in the external circuit (load)
$I_L = I_{LA} + I_{LB} = I_{GA} + I_{GB}$

Resistance in the external circuit R_L (load resistance)

EMF and internal resistance of the parallel operated voltage sources

The two parallel-operated power sources can be replaced by a single voltage source with the EMF $E_{A,B}$and internal resistance $R_{A,B}$

$$E_{A,B} = E_A \left(\frac{R_B}{R_A + R_B} \right) + E_B \left(\frac{R_A}{R_A + R_B} \right)$$

$$R_{A,B} = \frac{R_A \; R_B}{R_A + R_B}$$

Voltage at the terminals

With a direct parallel connection of the terminals of the two current outputs, the resistance of the connection line can be ignored, so that the following applies for the voltage at the terminals:

$$U_{Kl} = E_{A,B} - I_L \; R_{A,B}$$

or

$$U_{Kl} = \left(\frac{E_A}{R_A} + \frac{E_B}{R_B} - I_L \right) \frac{R_A \; R_B}{R_A + R_B}$$

Change of the terminal voltage

In addition to R_A and R_B, the terminal voltage is also dependent on E_A, E_B and I_L. A demand for change in the terminal voltage U_{Kl} at a constant load current I_L can be achieved by:

- *separate* change of E_A or E_B
- *combined* change of E_A and E_B

After a change of an individual EMF E_x by the amount ΔE_x the terminal voltage does not change by the entire amount ΔE_X [R_X / (R_A+R_B)].
Hence, with n identical internal resistances only by a factor of 1 / n.

A change in the terminal voltage is thus possible in steps smaller than ΔE_X. To change the terminal voltage by a certain amount, both voltage sources must therefore be changed by the same amount. In principle, this change in voltage can also be achieved by changing only one voltage source by twice the amount. The EMF of the voltage sources are then different. However, different EMFs cause an additional current (circulating current) within the parallel circuit.

Load current

For a given value of the load resistor R_L the following applies

$$I_L \frac{E_A \; R_B + E_B \; R_A}{R_A \; R_B + R_L \left(R_A + R_B \right)} = \frac{U_{Kl}}{R_L}$$

Distribution of the load current

The load current I_L is divided unequally across the voltage sources if the internal resistances are unequal. This may lead to overloading a power source.

The following applies for the distribution:

$$\frac{I_{LA}}{I_{LB}} = \frac{R_B}{R_A}$$

The total power available with unequal internal resistances is therefore less than the arithmetic sum of the two individual outputs.

Circulating current

The voltage sources operated in parallel form a current circuit in which with a difference $E_A - E_B \neq 0$ a current flows, called the circulating current I_{Kr}. The circuit current is determined solely by the data (EMF, R_x) of the two power sources, but not by the resistance of the load. This circulating current flows even when there is no load ($R_L = \infty$).

$$I_{Kr} = \frac{E_A - E_B}{R_A + R_B}$$

In the special case $R_A = R_B$ the circulating current is $I_{Kr} = ½ (I_{GA} - I_{GB})$

If E_A and E_B cannot be measured, the following applies for the circulating current:

$$I_{Kr} = I_{GA} - (I_{GA} + I_{GB})\left(\frac{R_B}{R_A + R_B}\right)$$

Superposition of load current and circulating current

The circulating current is superimposed in every voltage source on the load current. Where

$$(I_{LA} + I_{Kr}) + (I_{LB} - I_{Kr}) = I_{GA} + I_{GB} = I_{LA} + I_{LB} = I_L$$

The sign (direction) of I_{Kr} is determined by the higher EMF. If the EMF and I_{Kr} in are in the same direction (same polarity) in each voltage source, the currents I_{Lx} and I_{Kr} are added. In the voltage source with the smaller EMF, the EMF and I_{Kr} have opposite signs. Therefore, the circulating current I_{Kr} is subtracted from the respective load current I_{Lx}.

Currents in the voltage sources

For the distribution of the total current at the voltage sources, with $E_A > E_B$ (with $E_B > E_A$ the signs of I_{Kr} are reversed in the equation)

$$I_{GA} = I_L \frac{R_B}{R_A + R_B} + I_{Kr}$$

$$I_{GB} = I_L \frac{R_A}{R_A + R_B} - I_{Kr}$$

Controlling the current in the voltage sources

By appropriate adjustment of E_A and E_B, the value of of I_{GA} and I_{GB} and hence their ratio can be controlled so that the voltage sources are loaded in proportion to their nominal currents (the power rating). The ratio of the currents then coresponds to the ratio of the rated power.

$$I_{GA} = \frac{E_A (R_B + R_L) - E_B R_L}{R_A R_B + R_L (R_A + R_B)}$$

Example 1

$E_A = 122$ V; $R_A = 0.1\ \Omega$; $I_{NA} = 41.67$ A;
$E_B = 120$ V; $R_B = 0.05\ \Omega$; $I_{NB} = 83.33$ A;
$R = 1.3\ \Omega$
Calculated values: $E_{A,B} = 121$ V; $U_{Kl} = 118.5$ V

By increasing E_A and/or E_B in steps of $\Delta E_x = 1$ V, the increases in the terminal voltage and the associated changes the circulating current and the changes in the ratio of the currents in the power source to the nominal currents shown in Table 1 result.

Table 1

Δ_{EA} [V]	Δ_{EB} [V]	$\Delta_{EA,B}$ [V]	ΔU_{Kl} [V]	$I_{Kr\ B}$ [A]	I_{GA}/I_{NA}	I_{GB}/I_{NB}
0.00	0.00	0.000	0.000	13.33	0.95	0.47
+ 1.00	0.00	0.333	0.327	20.00	1.11	0.39
0.00	+ 1.00	0.666	0.653	6.67	0.79	0.55
+ 1.00	+ 1.00	1.000	0.979	13.33	0.95	0.47
0.00	+ 2.00	1.333	1.305	0.00	0.64	0.64

Example 2

$E_A = 122$ V; $E_B = 120$ V; $I_L = 100$ A; $R_A = 0,1\ \Omega$; $R_B = 0.05\ \Omega$
Calculated values:
$E_{A,B} = 120.67$ V; $U_{Kl} = 117.33$ V; $I_{Kr} = 13.33$ A; $R_{A,B} = 0.033\ \Omega$

Distribution of the load current
$I_{LA} = 33.33$ A; $I_{LB} = 66.67$ A
Sum $I_L = I_{LA} + I_{LB} = 33.33$ A + 66.67 A = 100 A

Current in the voltage sources
$I_{GA} = I_{LA} + I_{Kr} = 46,67$ A; $I_{GB} = I_{LB} - I_{Kr} = 53.34$ A
Sum $I_L = I_{GA} + I_{GB} = 46.67$ A + 53.34 A = 100 A

Continuation of the topic
Part 2 in Info Letter No. 10 / Part 3 in Info Letter No. 11

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The Excel programs used for the examples can be obtained from: ***www.a-eberle.de*** *(Download Center)*

The series will be continued.
We will gladly supply missing Info Letters at any time!

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