

# Info Letter No. 4

## Summation of Alternating Currents

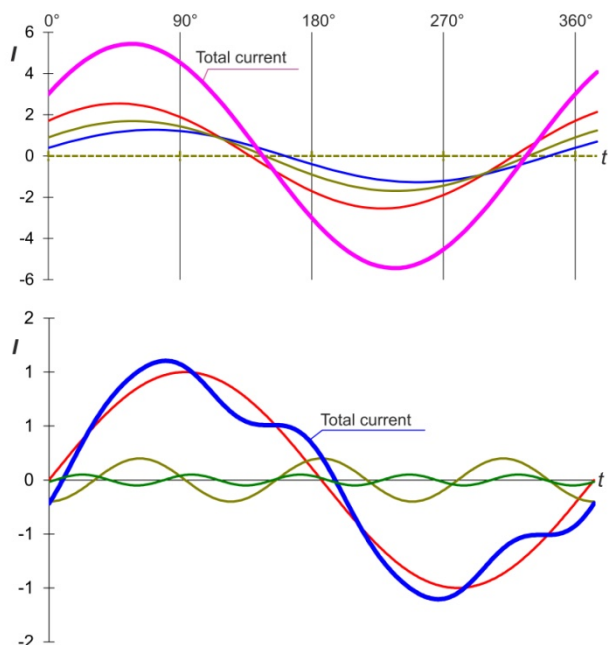
For the parallel connection of power sources, the principle of superposition applies, when all resources have a linear behaviour. This behaviour is the case with current transformers within the allowable load range and below a few thousand Hertz. It should be noted that in principle and device technically only simultaneous instantaneous values are added, not effective values!

Through the common load resistor  $R_L$  of parallel current transformers a current flows that corresponds to the arithmetic sum of the instantaneous values of all the currents. It is assumed that the current transformers provide superimposed currents and the frequency limits (highest order of the harmonics) are not exceeded.

The arithmetic sum of the instantaneous values is not only dependent on the amount, but also the sign of the individual currents, so that for the summation the polarity of the current sources must be considered. With alternating currents of the same direction of energy flow, the polarity is related to the secondary terminals S1 (k) and S2 (l) of the current transformer.

### Waveform of the total current

When summing equal frequency sinusoidal currents, the curve of  $i_\Sigma$  has the form of a sinusoidal curve, with currents of unequal frequency the waveform of the total current  $i_\Sigma$  is no longer sinusoidal.



**Figure 1** Waveform of the total current for the summation of three currents  
 a) with the same frequency,  
 b) with different frequencies

### Sum of the instantaneous values

For the three-phase three-wire network with the same frequency sinusoidal currents, the simultaneous instantaneous values are ideally zero.

#### Example 1

Given data:

$$I_1 = 3.85 \text{ A}, \varphi_1 = 41.0^\circ;$$

$$I_2 = 3.14 \text{ A}, \varphi_2 = 57.9^\circ;$$

$$I_3 = 2.65 \text{ A}, \varphi_3 = 35.0^\circ;$$

What is the arithmetic sum of the contemporaneous instantaneous values?

Result (calculation with Excel program E-1.7.1)

$$i_\Sigma = 3.85 \text{ A} \sin(0^\circ - 41.0^\circ) + 3.14 \text{ A} \sin(-120^\circ - 57.9^\circ) + 2.65 \text{ A} \sin(+120^\circ - 35.0^\circ)$$

$$i_\Sigma = -2.526 \text{ A} - 0.114 \text{ A} + 2.640 \text{ A} = 0$$

### Effective value of the total current

For the calculation of the effective value of the total current from the momentary values of current with an arbitrary frequency/waveform the following applies:

$$I_\Sigma = \sqrt{\frac{1}{T} \int_0^T (i_0 + i_1 + i_2 \dots + i_n)^2 dt}$$

$i_0 + i_1 + i_2 \dots + i_n$  Arithmetic total of the simultaneous current values

For the calculation only of the effective values of currents with different frequencies (e.g. equal value/ fundamental with harmonics) the following also applies:

$$I_\Sigma = \sqrt{I_0^2 + I_1^2 \dots + I_n^2}$$

$I_0$  Effective value of the direct current

$I_1$  Effective value of the fundamental

$I_n$  Effective value of the highest harmonic

### Measurement of the total current

To determine the sum  $i_\Sigma = i_1 + i_2 + i_3$  in the three-phase three-wire network both the terminals S1(k) and the terminals S2(l) of secondary windings of the three current transformers of the conductors L1, L2, L3 are connected (Holmgreen connection). A current which is the arithmetic sum of the instantaneous values of the three load currents flows through the load resistor; the sum in an earth fault-free network is in the ideal case equal to zero.

We take care of it.

### Measurement error

Because of the amplitude and angular error of current transformers, the total current  $I_{\Sigma}$  also differs from zero in the earth fault-free three-phase three-wire network.

#### Example 2

Given data:

$$I_1 = 4.60 \text{ A}, \varphi_1 = 23.0^\circ (\cos \varphi = 0.92); + 0.5 \%, + 0.5^\circ$$

$$I_2 = 4.50 \text{ A}, \varphi_2 = 12.3^\circ (\cos \varphi = 0.98); + 0.5 \%, + 0.5^\circ$$

$$I_3 = 3.80 \text{ A}, \varphi_3 = 19.0^\circ (\cos \varphi = 0.95); + 0.5 \%, + 0.5^\circ$$

How big is the variance of the total current from zero?

Results (calculation with Excel program E-1.7.1)

Instead of  $I_{\Sigma} = 0$ ,  $I_{\Sigma}^* = 0.07 \text{ A}$  is displayed.

This corresponds to 1.4 % of 5 A.  $\varphi_{\Sigma} = 15.6^\circ (\cos \varphi = 0.96)$

### Split-core current transformer

In this type of current transformer, an iron ring, on which a winding is applied, includes the three conductors of the cable. The magnetic fields of the three conductors induce a current in the winding whose strength is proportional to the arithmetic sum of the instantaneous values of the currents. This sum is ideally zero in an earth fault free network. - The influence described in the Holmgren circuit due to the measurement transformer error on the sum of the currents is not present in the split-core current transformer.

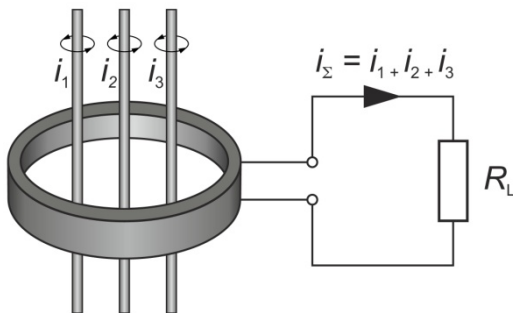


Figure 2 Split-core current transformer

### Earthing of cables with conductive coating

In order to eliminate the influence of the current  $I_{\text{Sheath}}$  flowing in the cable sheath over  $C_{LE}$ , the earth conductor from the termination of the cable must pass through the iron core of the split-core current transformer parallel to the cable, however, but in the opposite direction to earth. As a result, the opposing fields flowing in the cable sheath and current flowing in the earth conductor  $I_{\text{Sheath}}$  cancel out. It is now determined by the sum of  $i_1 + i_2 + i_3$ . - In order to achieve this, the termination of the cable must be isolated from earth.

### Summation current transformers

This type of current transformer is used for the summation of equal frequency alternating currents, which can

have any phase angle. The summation current transformer has a ring core, which carries multiple input coils (primary windings), and the secondary winding.

In equally loaded networks with the same operating voltage, the overall performance of n junctions can be determined with a summation current transformer and a single power measurement equipment.

#### Example 3

Given data:

The values of the currents in L1 in the three branches A, B, C of a three-phase three-wire network.

$$I_{1A} = 3.6 \text{ A}, \varphi_{1A} = 15^\circ;$$

$$I_{1B} = 2.8 \text{ A}, \varphi_{1B} = 45^\circ;$$

$$I_{1C} = 1.7 \text{ A}, \varphi_{1C} = 32^\circ;$$

What is the effective value of the total current  $I_{\Sigma}$  ?

Result (calculation with Excel program E-1.8.2)

$$I_{\Sigma} = 7.88 \text{ A}; \varphi_{\Sigma} = 28.9^\circ; \cos \varphi_{\Sigma} = 0.875$$

Results for the total active power in the branches A, B, C at the same operating voltage of 400 V is

$$P_A = 1391 \text{ W}, P_B = 792 \text{ W}, P_C = 577 \text{ W},$$

$$P_A + P_B + P_C = 2760 \text{ W};$$

with the values of  $I_{\Sigma}$  this also gives

$$P_{\Sigma} = 2760 \text{ W}.$$

### Total current with a single pole earth fault

The load currents flowing in the conductors are the earth currents flowing to earth superimposed on the phase-earth capacitances. For the same values of the three phase-earth capacitances of a conductor, the sum of these "capacitive" earth currents in earth fault free operation is zero, so that in this condition the sum of the load currents and the earth currents is zero.

In a single-pole saturated earth fault in a circuit, the voltage and thus the current between the conductor and earth is zero, so that the sum of the conductor currents due to  $I_1 + I_2 + I_3 = 0$ , e.g. with an earth fault on conductor L1,  $I_E = 0 + I_{2E} + I_{3E} \neq 0$  for the earth current and thus only the currents of the earth fault free conductor flowing through the two phase-earth-capacitances give the total.

### Active component of the earth current

When the phase-earth-capacitances are the same and the same phase-earth voltages, the earth current has a angular difference of  $90^\circ$  compared to the zero sequence voltage. With different values and/or when there is a significant resistance in series with the phase-earth capacitance, the angular difference is no longer exactly  $90^\circ$ . The deviation from  $90^\circ$  may be a few degrees in either direction, so that the earth current then also has an active component. This value is normally very small; the sign is a function of the said conditions.

We take care of it.

**Example 4**

Three-phase three-wire network with earth fault compensation, earth fault on conductor L1. Voltages at the measurement location / measurement equipment:

$$\underline{U}_{12} = 20.243 e^{j30^\circ} \text{ kV};$$

$$\underline{U}_{23} = 20.168 e^{-j90^\circ} \text{ kV};$$

$$\underline{U}_{31} = 20.393 e^{j150^\circ} \text{ kV};$$

Phase-earth capacitances:

$$C_1 = 5.9 \mu\text{F}; C_2 = 6.0 \mu\text{F}; C_3 = 6.2 \mu\text{F} \text{ (asymmetry 3\%);}$$

Conductor derivations (conductor-earth):

$$R_1 = 100 \text{ k}\Omega; R_2 = 102 \text{ k}\Omega; R_3 = 99 \text{ k}\Omega \text{ (asymmetry 2\%);}$$

$$\text{Earth fault on conductor L1: } Z_F = 23 \Omega;$$

Earth fault suppression coil:

$$\text{Impedance } Z_{NE} = 173 e^{j86^\circ} \Omega$$

(Inductance  $L = 0,55 \text{ H}$ ; Copper resistance  $R = 9 \Omega$ ;

Iron losses  $R = 7000 \Omega$ );

What is the earth current in the individual conductors to earth and what is the geometric sum of these currents?

**Results**

(Calculation with program E-1.7.2; see annex)

Phase-earth voltages:

$$\underline{U}_{23} = 0.14 e^{j8^\circ} \text{ kV};$$

$$\underline{U}_{31} = 20.113 e^{-j150^\circ} \text{ kV};$$

$$\underline{U}_{31} = 20.283 e^{j150^\circ} \text{ kV};$$

$$Z_{1E} = 513.7 e^{-j90^\circ} \Omega;$$

$$Z_{1E} = 540.0 e^{-j90^\circ} \Omega;$$

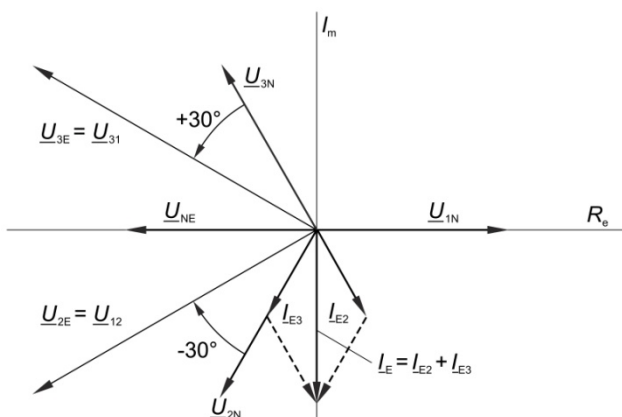
$$Z_{3E} = 522.1 e^{-j90^\circ} \Omega;$$

$$I_{1E} = 6 e^{j11^\circ} \text{ A};$$

$$I_{1E} = 38 e^{-j63^\circ} \text{ A};$$

$$I_{1E} = 39 e^{-j123^\circ} \text{ A};$$

$$I_E = 66 e^{-j88^\circ} \text{ A (active component: 2 A)}$$



**Figure 3** Earth current  $I_E$  with earth fault on L1

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The Excel programs used for the examples can be obtained from:

[www.a-eberle.de](http://www.a-eberle.de)

(Download Center)

The series will be continued.

We will gladly supply missing Info Letters at any time!

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