We take care of it.

a-eberle

Info Letter No. 8

Apparent power and reactive power in three-phase networks

Differences in measurement results

The range of electromechanical and electronic measuring devices in analog - and digital technology for the measurement of output values is extensive. A comparison of the technical documentation of various manufacturers shows that for output values no, or different definitions are given. This leads to no longer comparable measurement results for distorted sine waveform of the alternating quantities and/or significantly unbalanced loaded three-phase networks, especially for the reactive power measurement. But even with an undistorted sinusoidal waveform, with an unbalanced load the measuring equipment from different manufacturers and all things being equal can display different readings These differences may indeed be ignored with low demands on the accuracy and at approximately ideal operating conditions, but not with a significant deviation from it.

Different results can be expected and actually found, because on the one hand a uniform and generally applicable definition of the definitions for the measurement of apparent power and reactive power has not yet been implemented, and on the other, the state of the measuring technology nowadays enables an investigation of all the allowed instantaneous values of the voltages and currents sizes without special additional effort.

For a sensible treatment of the variety of these variables, the manufacturers and users of the measurement instruments must therefore be provided with knowledge of the definition and significance of the values. There must be clarity before the measurement about what is actually being measured (and why). What measurement equipment fulfill this task and what falsifying factors can affect the measurement result?

Apparent power in three-phase networks

In the energy conversion at the consumer no losses will occur if the consumer is constructed as a symmetric load circuit with linear active resistances and in each phase between the time course of voltage and current there is direct proportionality. All of these conditions must be fulfilled; balanced voltages and currents are however not a condition.

Measurement equipment for apparent power

The crucial questions for the efficiency of a resource are therefore: What is the relationship between the apparent and the effective converted into another form of energy, such as electrical power and how is the apparent power quantified?

Logically, as for the apparent power in the two-wire AC network, in the three-wire three-phase network the product of the collective voltage U_Σ and the collective conductor current I_Σ is identified as the collective apparent power S_Σ.

$$S_\Sigma = U_\Sigma \cdot I_\Sigma \qquad (1)$$

In accordance with DIN 40110 Part 2 [1] for a three-phase three-wire network the following applies

$$S_\Sigma = \sqrt{U_{10}^2 + U_{20}^2 + U_{30}^2} \cdot \sqrt{I_1^2 + I_2^2 + I_3^2} \qquad (2)$$

or when using the phase-phase voltages

$$S_\Sigma = \sqrt{\frac{1}{3}\left(U_{12}^2 + U_{13}^2 + U_{23}^2\right)} \cdot \sqrt{I_1^2 + I_2^2 + I_3^2} \qquad (3)$$

and for a three-phase four-wire network

$$S_\Sigma = \sqrt{U_{10}^2 + U_{20}^2 + U_{30}^2 + U_{N0}^2} \cdot \sqrt{I_1^2 + I_2^2 + I_3^2 + I_N^2} \qquad (4)$$

The device-related reproduction of equations (2), (3) and (4) is not technically/economically feasible with electromechanical sensors, so in the past (based on $P = P_1 + P_2 + P_3$), in general the equation $S = U_{10} I_1 + U_{20} I_2 + U_{30} I_3$ was and is still frequently used in digital measurement equipment.

It had already been conclusively proven by E. Weber [2] in 1929 that this equation for the apparent power in three-phase networks is not accurate. A comprehensive and coherent theory for the definition of this measure is provided by a work of J. Brenner [3] and DIN 40110 Part 2.

Reactive power in three-phase networks

For the collective reactive power Q_Σ in a three-phase network, which is initially only a mathematical parameter derived from S_Σ and P_Σ, the agreement applies that the values of the active power and reactive power are the two orthogonal components of the apparent power:

$$Q_\Sigma = \sqrt{S_\Sigma^2 - P_\Sigma^2} = S_\Sigma \sin \varphi \qquad (5)$$

The resulting calculated reactive power includes all losses caused by reactances, non-linear resistances (harmonics) and asymmetry of the load. Reactive power according to this definition is thus already claimed by a load of unequal active resistances (Example 2)!

Measurement equipment for reactive power

As in the case of active power in the past for the reactive power, the equation

$Q^* = Q^*_{L1} + Q^*_{L2} + Q^*_{L3}$

with the respective phase power

$Q^*_i = U_i \cdot I_i \cdot cos(90° - \varphi_i)$

is generally used.

With electro-mechanical measurement equipment that was already the only viable solution possibility and had shaped the reactive power measurement for a long time. The questionable nature of this reactive power definition is also discussed by E. Weber [3] in detail.

Because of the undefined effect of the waveform in non-sinusoidal alternating values and because of the additional effect of the unbalanced phase-phase voltages on the measurement result, such reactive power measurements generally provide only uncertain readings.

Conditions in the supply networks

In the high and medium voltage networks of power companies, the load of the three phases is approximately symmetrical and the shape of the curve of the alternating values is sinusoidal. The reactive power is therefore still primarily measured using the equation

$Q^* = Q^*_{L1} + Q^*_{L2} + Q^*_{L3}$

because of the value thus measured of the measured value for Q, under these conditions differs only slightly (Example 2).

Only for sinusoidal alternating values, symmetrical voltages, symmetrical currents and the same phase angle do the equations for Q_Σ and Q^* give the same results, otherwise the results are not equal.

For these reasons and because of the lack until the eighties of the last century of technical equipment simulation possibilities for the equation (5) until then there was no incentive and no obligation to deal with the definition and measurement of reactive power in three-phase networks. This situation has until today, especially in energy distribution, hardly changed at all for the practitioner.

Measurement of the phase reactive power with sinusoidal AC values

With simple measurement equipment, the equation

$Q^* = U \cdot I \cdot sin\ \varphi$

cannot be reproduced because the multiplication of u and i and the subsequent integration always produces the result

$U \cdot I \cdot cos\ \varphi$

but not the required result

$U \cdot I \cdot sin\ \varphi$.

because

$sin\ \varphi = cos(90° - \varphi)$

the reactive power with sinusoidal AC values can be determined by measurement just as the active power, if

prior to the multiplication of the instantaneous values of voltage and current, the voltage is shifted by 90° against the current.

This shift can be achieved in each phase in two ways:

With digital measurement equipment, the respective sample of the voltage is not multiplied with the corresponding, but with the $\pi/2$ (corresponding to 90°) offset sample of the current.

$$Q* = \frac{1}{N}\sum_{k=l}^{N} u(t_k) \cdot i(t_{k+N/4}) \qquad (6)$$

N Number of samples per period
k Index of sample values

For analog measurement equipment in the three-phase network, such a displacement can be achieved by using the respective vector of the vertical star voltage vector of the phase-phase voltage.

Through the connection of the corresponding phase-phase voltages to the measurement equipment, a simple frequency-independent shift of the voltage against the current $(90° \pm \varphi)$ is achieved. The change in angle is, however, only exactly 90° when the phase-to-phase voltages are symmetric. This also applies for equations (7) and (8).

For sinusoidal alternating values the equation for Q^* can be specified in the following form (for analog measurement equipment):

$Q^* = 1/\sqrt{3}\ [U_{23} \cdot I_1 \cdot cos(\varphi_{23} - \psi_1) + U_{31} \cdot I_2 \cdot cos(\varphi_{31} - \psi_2) + U_{12} \cdot I_3 \cdot cos(\varphi_{12} - \psi_3)]$ (7)

φ_{ij} Angle of the voltage vectors at the real axis
ψ_i Angle of the current vectors at the real axis

For the three-wire three-phase network, i.e. under the condition $\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$ and connection of the measurement equipment in accordance with DIN 43807 (virtual neutral point, two measurement stations), equation (6) can also be specified in the following form:

$Q^* = \sqrt{3}\ [U_{10} \cdot I_3 \cdot cos(\varphi_{10} - \psi_3) - U_{30} \cdot I_1 \cdot cos(\varphi_{30} - \psi_1)]$ (8)

In this measurement method, only the reactive power is determined that is caused by the reactances of the individual phases. The reactive power caused by the unbalanced load on the network is not determined.

Effect of non-sinusoidal alternating values

The results with non-sinusoidal alternating values determined for Q by equation 5 do not match the results according to equation 6. The following still applies $Q^a < Q$.

The measured value obtained from the sample values from equation 6 is that same as the calculated value using equation 9. For this a sufficiently large number N of samples and a precise band limit is specified and other technical error causes are not taken into account.

$$Q^a = \sum_{\nu=1}^{\infty} U_\nu I_\nu \cos\left(\nu\, 90° - \varphi_\nu\right) \qquad (9)$$

φ_ν *Phase angle between the harmonics of the voltage and current of the ordinal number*.

or in the detailed form
$Q^a = U_1 \cdot I_1 \cdot cos\ (1\cdot90° - \varphi_1)\ ... + U_\nu \cdot I_\nu \cdot cos\ (\nu\cdot90 - \varphi_\nu\ )$

As for the active power, only the same frequency harmonics of the voltage and current for Q^a have a value greater than zero. The integral (the arithmetic mean) over the time course of the instantaneous value of the reactive power for a period with non-frequency harmonics gives the value zero.

Reactive power of the fundamentals

If only the current is non-sinusoidal, that voltage however has a sinusoidal course (thus no harmonics) with Q^a the reactive power of the fundamental is measured, because then the factor $U_{n\sqrt{}}$ is equal to zero, and hence the products of the harmonics are also equal to zero.

$$Q^a = U_1 I_1 \cos\left(90° - \varphi_1\right) + 0...+0 \qquad (10)$$

If the individual harmonics of the voltage do not have the same frequency as those of the current, the measurement result corresponds to the fundamental reactive power.

Example 1

Given: a non-sinusoidal voltage with the fundamental U_1 = 230 V and the harmonics U_3 = 35 V and U_5 = 16 V. Further, a non-sinusoidal current with the fundamental I_1 = 4.60 A and the harmonics I_3 = 0.70 A and I_5 = 0.32 A. Phase angle φ_1 = 37°; φ_3 = 42°; φ_5 = 26°.

Active power
P = 230 V · 4,60 A · cos 37° + 35 V · 0,70 A · cos 42° + 16 V · 0,32 A · cos 26°
= 845 W + 18 W + 5 W = 868 W

Apparent power
S = 233.2 V 4.66 A = 1088 VA

Reactive power
Q^a = 230 V · 4,60 A · cos (1·90° - 37°) +35 V · 0,70 A · cos (3·90° - 42°) +16 V · 0,32 A · cos (5·90° - 26°)
= 637 Var - 16 Var + 2 Var = 623 Var

$$Q = \sqrt{(S^2 - P^2)} = 656\,\text{Var}$$

Difference of the reactive power values
Q^a - Q_Σ = 623 Var - 656 Var = - 33 Var;
percentage deviation of Q_Σ: - 5 %

Fundamental reactive power
Q_1 = 230 V · 4,60 A · cos (90° - 37°) = 637 Var

Sign of the reactive power

As with the active power measurement, unlike Q_Σ, the measured values of Q^a have a sign that is dependent on the load type (inductive or capacitive) and the direction of the energy flow. In energy terms and with an inductive reactance of the load, the phase angle φ has a positive sign, and thus also the value of Q^a.

The sign is negative with capacitive reactance of the consumer. If the direction of the energy flow reverses, the sign also changes.

Example 2
For the calculations (with program E-1.4.1; retrievable from **www.a-eberle.de** (Download Center)
sinusoidal alternating values are used.

Three-phase three-wire network

	Example A	Example B
U_{12}	104,5 V	101,2 V
U_{23}	103,9 V	100,6 V
U_{31}	104,9 V	101,9 V
$\neq U$	0,5 %	0,8 %
I_1	4,3 A (28 W)	4,0 A
I_2	4,4 A (26 W)	4,4 A
I_3	3,8 A (34 W)	3,6 A
$\neq I$	8,1 %	11,8 %
φ_{10}	-4,4° (cos φ = 0,997)	23° (cos φ = 0,92)
φ_{20}	3,2° (cos φ = 0,998)	32,9° (cos φ = 0,84)
φ_{30}	1,2° (cos φ = 1,000)	34,1° (cos φ = 0,83)
P_Σ	748 W	609 W
S_Σ	751 W	708 W
ΔS^*	-0,2 % (S^* = 749 VA)	-0,4 % (S^* = 706 VA)
Q_Σ	61 VAr	361 VAr
ΔQ^a	-99 % (Q^a = 0,7 Var)	-2,5 % (Q^a = 352 Var)

We take care of it.

Three-phase four-wire network

	Example A	Example B
U_{1N}	228,2 V	227,6 V
U_{2N}	231,2 V	230,9 V
U_{3N}	230,0 V	231,8 V
$\neq U$	0,7 %	0,7 %
I_1	12,5 A (18 W)	12,4 A
I_2	10,5 A (22 W)	10,5 A
I_3	9,3 A (25 W)	9,4 A
$\neq I$	17,6 %	17,0 %
φ_{10}	0,0° (cos φ = 1,000)	22,6° (cos φ = 0,92)
φ_{20}	1,0° (cos φ = 1,000)	20,3° (cos φ = 0,94)
φ_{30}	0,5° (cos φ = 1,000)	27,4° (cos φ = 0,89)
P_Σ	7440 W	6793 W
S_Σ	7573 W	7604 W
ΔS^*	-2 % (S^* = 7440 VA)	-3 % (S^* = 7403 VA)
Q_Σ	1416 Var	3416 Var
ΔQ^a	-98 % (Q^a = 27 Var)	-15 % (Q^a = 2918 Var)

Q^a Reactive power, connection and measurement equipment in accordance with DIN 43807; see equation 7 and 8

φ_{10} Angle between I_1 and U_{10}

$\neq$ [%]   Asymmetry (negative sequence / positive sequence)

ΔS^* percentage deviation of S_Σ

ΔQ^a percentage deviation of Q_Σ

References:

[1] DIN 40 110: Wechselstromgrößen, Teil 1, Zweileiter - Stromkreise, Beuth Verlag GmbH, Berlin und Köln 03.94 und DIN 40 110: Wechselstromgrößen, Teil 2, Mehrleiter - Stromkreise, Beuth Verlag GmbH, Berlin und Köln

[2] Weber, E.: Zur Definition von "Scheinleistung", „Blindleistung" und Leistungsfaktor, Elektrotechnik und Maschinenbau, April 1929 Seite 277...284.

[3] Brenner, J.: Scheinleistung im Mehrphasensystem, Sonderdruck Metrawatt GmbH Juni 1992 und Elektrische Leistung im Mehrphasensystem, 1998 (retrievable from *www.a-eberle.de* (Download Center)

[4] Karger, H.: Messungen in Netzen der elektrischen Energietechnik, Pia-Verlag Nürnberg, 1994

[5] Karger, H.: *messen + regeln* in Starkstromnetzen Teil 1; A. Eberle GmbH & Co. KG 2003