– Definition of power measurements according to the standards –
– DIN 40110-2 und IEEE 1459 –

There is a wide range of electronic measuring devices for the digital measurement of power quantities available today. A comparison between the various producers shows, that different measurement results may occur, especially in the case of reactive power measurement with a distorted sinusoidal shape and/or three-phase networks with a clearly unbalanced load.

This can cause problems in the design and operation of installations up to overloads and failures.

The power measurement in multi-wire circuits is defined in the standards DIN 40110-2 (Germany) and IEEE 1459 (International). These fundamentals serve the basis for power calculations in modern measuring instruments.

All devices of the A-Eberle product range use the calculation method according to DIN 40110-2.

In the following, the basic definitions of active, apparent and reactive power are first described and then illustrated with examples of the individual types of reactive power.

In this context, the differences between power measurement in the single-phase and multi-phase system as well as special features in the 4- or 3-wire network are discussed.

Power definitions

Active power is the only power that can be calculated directly from the instantaneous values of voltage $u(t)$ und current $i(t)$. The instantaneous power $p(t)$ for each sampling point t is calculated as the product of voltage and current.

$$p(t) = u(t) \cdot \mathrm{i(t)} \tag{1}$$

By integration of instantaneous power $p(t)$ within the time interval $T = t_2 - t_1$ the total energy E is calculated. The time interval T is freely selectable and e.g. can be set to a half net frequency period (for TRMS fault records) or 15 minutes (for energy measurement).

$$E = \int_{t_1}^{t_2} p(t)\,dt \tag{2}$$

If energy is transported from a source to a consumer (e.g. light bulb, motor) and absorbed there, it is referred to active energy. In case of energy storage at the consumer (e.g. in capacitors) and the later return transport to the source, the energy is called reactive energy.

Therefore, power is defined as energy transport per time from a source to a load. The part of the power that is returned to the source within the considered time interval is called reactive power Q, while the part of the power which is converted at the load into other forms of power (e.g. thermal power, light, mechanical power) is called active power P.

This examination shows clearly that the calculated value for active and reactive power is not only dependent on the voltage and current curve, but also on the considered time interval T.

Apparent power

Apparent power is often used for the design of electrical equipment and can be derived from the TRMS[1] values of voltage and current in single-phase systems and therefore has no sign.

$$S = U_{trms} \cdot I_{trms} \tag{3}$$

The collective apparent power S_Σ of a three-wire system is calculated from the values of collective voltage U_Σ and collective current I_Σ, which are defined in DIN 40110-2.

$$S_\Sigma = U_\Sigma \cdot I_\Sigma \tag{4}$$

The meaning of these variables is explained in more detail in the section "Reactive power in the three-phase system".

Active power

As mentioned before, active power P represents the part of the power that is transported to the load within the considered time interval $T = t_2 - t_1$ and does not return to the source. The active power is defined as the mean value of instantaneous power $p(t)$ and has an algebraic sign, which indicates the direction of energy flow depending on the counting arrow system.

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\,dt = \frac{E}{T} \tag{5}$$

In contrast to apparent power, active power represents a temporal mean value, which allows the collective active power P_Σ of a three-wire system to be calculated as the sum of the active powers of the individual phases.

$$P_\Sigma = P_{L1} + P_{L2} + P_{L3} \tag{6}$$

Reactive power

The reactive power represents the part of the power that is transferred from the source to the load and back again within the considered interval T and is generally derived from the relationship:

$$Q = \sqrt{S^2 - P^2} \tag{7}$$

[1] TRMS – true root mean square = ac and dc share

Reactive power represents a derived quantity from apparent and active power and is never calculated directly.

The total reactive power of an electrical energy system Q_Σ consists of 5 different types of reactive power, which can be summarized by square addition:

$$Q_\Sigma = \sqrt{Q_{h1}^2 + {Q_{hn}}^2 + Q_d^2 + Q_m^2 + Q_u^2} \qquad (8)$$

Table 1: Overview reactive power

Quantity	Component	Cause
Q_{h1}	Fundamental displacement reactive power	Phase shift φ_{h1}
Q_{hn}	Harmonic displacement reactive power	Phase shift φ_{hi}
Q_d	Distortion reactive power	Current harmonics
Q_m	Modulation reactive power	Power fluctuations
Q_u	Unbalance reactive power	Unbalanced three-phase system load

For the calculation of reactive power, a general distinction must be made between single-phase and three-phase systems

Reactive power in single-phase systems

In single-phase systems, the reactive power types Q_{h1}, Q_{hn}, Q_d and Q_m may occur. Unbalance reactive power can only be calculated three-phase systems.

Displacement reactive power

The displacement reactive powers Q_1 and Q_h are caused by the phase shift φ_i between voltage and current of equal-frequency signal components.

$$Q_{h1} = U_{h1} \cdot I_{h1} \cdot \sin\varphi_{h1} \qquad (9a)$$

$$Q_{hn} = \sqrt{\sum_{i=2}^{n} (U_{hi} \cdot I_{hi} \cdot \sin\varphi_{hi})^2} \qquad (9b)$$

The component Q_1 describes the displacement reactive power of the fundamental oscillation of voltage and current. Q_{hn} represents the sum of the displacement reactive powers of all harmonic components.

The examples A and B in Table 2 show the influence of the displacement reactive power on the current carrying capacity of the conductors in the grid.

In both cases, an active power (average power over a grid frequency period) of $P = 1{,}012\ kW$ is consumed. In Example A, voltage and current are in phase ($\varphi_{h1} = 0°$).

The time curve of the instantaneous power in Figure 1 shows that in this case the power $p_A(t)$ is always positive and therefore there is no power swing between source and load. No reactive power occurs.

Table 2: Example for displacement reactive power

Quantity	Example A	Example B
U_{h1}	230 V	230 V
I_{h1}	4,4 A	8,8 A
φ_{h1}	0 °	60 °
P	1012 W	1012 W
S	1012 VA	2024 VA
Q_{h1}	0 Var	1752 VAr

In example B the same active power is transmitted with a phase angle of $\varphi_{h1} = 60°$. Due to this shift, a power flow from the load to the source (negative instantaneous power) occurs in the first part of the grid frequency period, which must be compensated later by a higher power flow from the source to the load in order to transmit the identical active power P . In the given case, this can only be achieved by doubling the phase current from 4.4 A in example A to 8.8 A in example B.

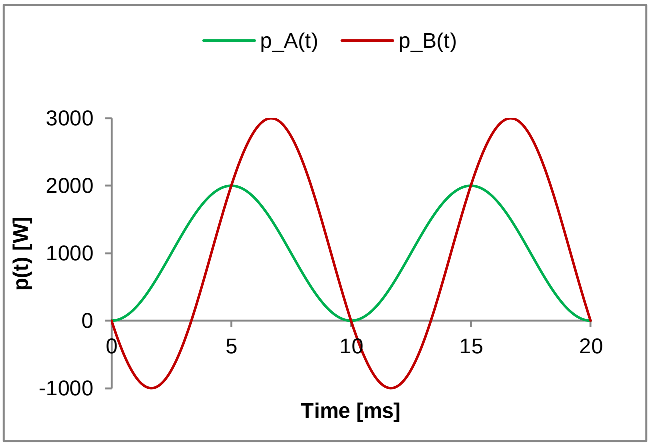

Figure 1: Instantaneous power of one period with displacement reactive power

The higher current flow leads to a greater stress on the equipment in the network. This additional stress can be described by the increase of the apparent power (from 1024 VA to 2048 VA) as well as the build-up of a reactive power of $Q_{h1} = 1745$ VAr.

The displacement reactive power is the only reactive power type with a sign that describes the type of reactive power. In consumer counting system, the negative reactive power is defined as capacitive and the positive reactive power as inductive.

Distortion reactive power

In addition to the temporal shift of voltage and current, the combination of non-equal-frequency oscillation parts also leads to the formation of reactive power, since in this case the average value of the resulting energy over the signal period is always zero.

According to equation 5, no active power can be produced although the transmission line is stressed, which leads to the formation of reactive power by definition.

Example C from Table 3 shows a sinusoidal voltage with a fundamental current of 4.4 A and a 5th harmonic current with an RMS value of 2 A.

The harmonic current does not contribute to the active power, but causes the instantaneous power to oscillate around the ideal power curve (Figure 2).

Table 3: Example distortion reactive power

Quantity	Example A	Example C
U_{h1}	230 V	230 V
I_{h1}	4,4 A	4,4 A
I_{h5}	0 A	2 A
φ_{h1} φ_{h5}	0 ° 0 °	0 ° 0 °
P	1012 W	1012 W
S	1012 VA	1112 VA
Q_d	0 VAr	460 VAr

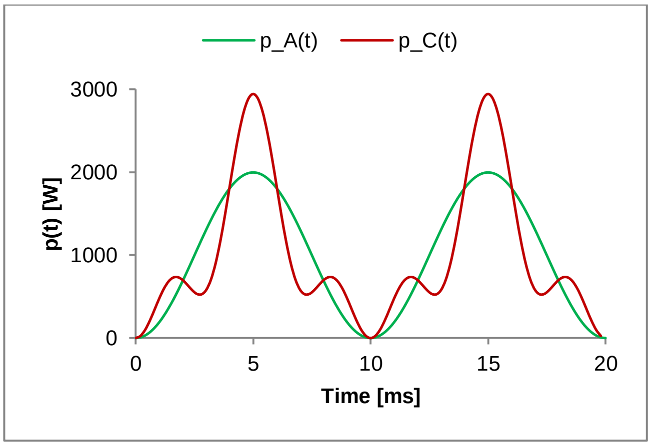

Figure 2: Instantaneous power of one period with distortion reactive power

The resulting distortion reactive power Q_d can be calculated from the apparent power S, which results from the product of grid voltage and the geometric sum of the partial currents I_{ges} .

$$I_{ges} = \sqrt{I_{h1}^2 + I_{h5}^2} = 4{,}833\ A \quad (10)$$

$$S = U_{h1} \cdot I_{ges} = 1112\ VA \quad (11)$$

$$Q_d = \sqrt{S^2 - P^2} = 460\ VAr \quad (12)$$

Displacement and distortion reactive power

Considering realistic voltage and current curves, a combination of different types of reactive power occurs. The individual reactive power components cannot be combined directly, instead they must be added squarely according to equation 8.

Figure 3 shows a combination of the reactive power from example B (Q_1) and example C (Q_d). The total reactive power can be calculated via quadratic addition of Q_1 and Q_d according to Equation 13.

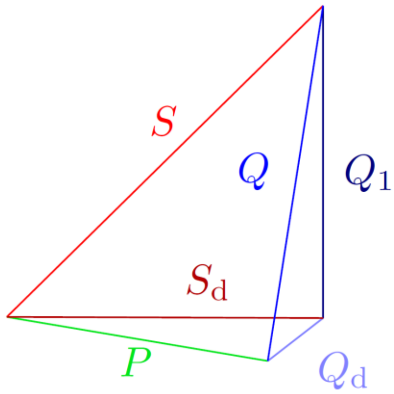

Figure 3: A combination of displacement and distortion reactive power

$$Q = \sqrt{Q_d^2 + Q_{h1}^2} = 1811\ VAr \quad (13)$$

$$S = \sqrt{P^2 + Q^2} = 2075\ VA \quad (14)$$

Modulation reactive power

While displacement and distortion reactive power are measurable with minimum observation interval of half a grid frequency period, modulation reactive power can only be detected at longer measuring intervals.

Thereby, the observation time required for correct measurement of modulation reactive power is strongly dependent on the characteristics of the equipment to be analyzed (e.g. fan heater, periodically controlled heating, elevator).

The example in table 4 shows the operation of a fan heater with a pulse frequency of 1 Hz (modulation period duration 1 s).

In the first part of the period (500 ms) the heater and fan motor operate simultaneously and the current consumption is 15 A. During the second part, only the fan is still active, reducing the current consumption to 5A. To simplify matters, the current is assumed to be sinusoidal and in phase with the voltage, so that neither displacement nor distortion reactive power is produced.

Table 4: Example fan heater

Phase	Quantities
Heater + Fan	$T_A = 500\,ms$ $P_A = 3450\,W$ $I_A = 15\,A$ $Q_{h1,A} = Q_{d1,A} = 0\,VAr$
Fan	$T_B = 500\,ms$ $P_B = 1150W$ $I_B = 5\,A$ $Q_{h1,B} = Q_{d1,B} = 0\,VAr$
Modulation period	$T_{mod} = 1000\,ms$
Voltage	$U_{trms} = 230\,V$

In this case, the result of the power measurement strongly depends on the chosen measurement interval. In case a smaller interval than the modulation period is selected (e.g. T = 300 ms), the power values will fluctuate continuously as the mean value of the power oscillation changes within the selected time window (see Figure 4).

Only when the measurement interval is chosen as a multiple of the modulation period T_{mod}, constant power values are generated.

It is noticeable that the calculated active power P_{total} differs from the apparent power S_{total}, although no displacement or distortion reactive power does occur as mentioned before.

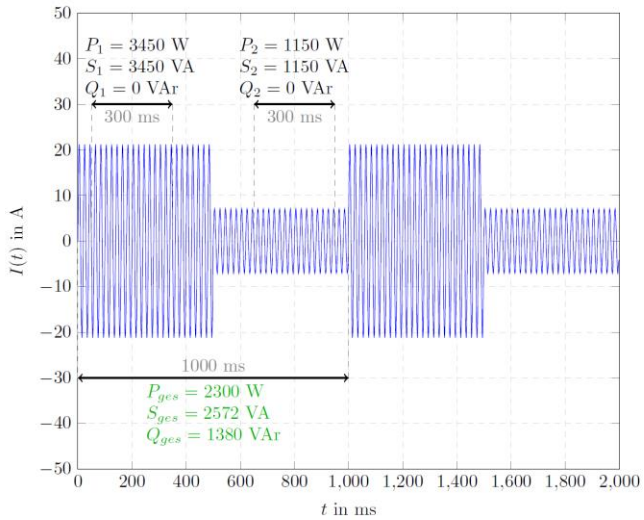

Figure 4: Current flow of a pulsed fan heater

This difference can be explained by the modulation reactive power Q_m. This type of reactive power is characterized by the fact that the transmission line between source and load is ideally used in the individual modulation phases, but not under consideration of the whole measurement cycle.

In the example in Table 4, an ideal transmission would take place at a constant power of $P = 2300\,W$. Due to the power fluctuation as a result of modulation, the power line is loaded heavier in the first section and loaded weaker in the second section, which leads to an increase in overall transmission losses. For example, a constant power transmission could be achieved by installing an energy storage system at the connection point of the load.

In order to determine the modulation reactive power, at first the active power P_{total} in the considered interval of $1000\,ms$ is calculated from the product of the arithmetic mean of the RMS current values of the two modulation sections and the RMS voltage value (see equation 15c).

As defined, the apparent power S_{total} must be calculated from the RMS values of voltage and current of the entire measuring cycle according to equation 15d. The geometric difference between apparent and reactive power then yields to the reactive power Q_m according to equation 15e.

$$\bar{I} = \frac{I_A + I_B}{2} = 10\,A \tag{15a}$$

$$I_{trms} = \sqrt{\frac{1}{m}\sum_{n=1}^{m} I_n^2} = 11{,}185\,A \tag{15b}$$

$$P_{total} = U_{trms} \cdot \bar{I} = 2300\,W \tag{15c}$$

$$S_{total} = U_{trms} \cdot I_{trms} = 2572\,VA \tag{15d}$$

$$Q_m = \sqrt{S_{total}^2 - P_{total}^2} = 1150\,VAr \tag{15e}$$

Power factor and $\cos\varphi$

An ideal energy transmission system is characterized by the fact that the electrical power output from the power source is completely consumed at the load. In this case, the apparent power (with which the transmission system is loaded) corresponds to the converted active power at the consumer.

To describe the quality of a non-ideal system, the power factor λ is used as a quotient of active and apparent power.

$$\lambda = \frac{|P|}{S} \tag{16}$$

In the ideal case, this factor has the value λ=1 and is reduced to 0 as the reactive power component increases.

The value $\cos\varphi$, which is often mentioned in context with power factor λ is only valid in networks whose voltage and current curves are ideally sinusoidal and symmetrical. Therefore ist is used exclusively to describe the displacement reactive power of the fundamental oscillations.

Reactive power in three-phase systems

All calculation methods mentioned so far are defined for single-phase systems and cannot be applied directly to three-phase systems. The only power values that can be determined directly from the sum of the individual phases are active power, displacement reactive power and distortion reactive power.

Since the single-phase apparent power S is not a vector, the apparent power of a three-phase system cannot be calculated by (scalar or vectorial) addition of the apparent powers of the individual phases.

For this reason, the collective apparent power S_{Σ} is defined according to DIN40110-2 as the product of collective voltage U_{Σ} and collective current I_{Σ}.

$$S_{\Sigma} = U_{\Sigma} \cdot I_{\Sigma} \tag{17}$$

For the calculation of the collective values, the following differentiation is made between a 4-wire and 3-wire system:

4- wire system:

$$U_{\Sigma\,4L} = \sqrt{U10_{trms}^2 + U20_{trms}^2 + U30_{trms}^2} \tag{17a}$$

$$I_{\Sigma\,4L} = \sqrt{I1_{trms}^2 + I2_{trms}^2 + I3_{trms}^2 + IN_{trms}^2} \tag{17b}$$

3- wire system:

$$U_{\Sigma\,3L} = \sqrt{\frac{1}{3}(U12_{trms}^2 + U23_{trms}^2 + U31_{trms}^2)} \tag{17c}$$

$$I_{\Sigma\,3L} = \sqrt{I1_{trms}^2 + I2_{trms}^2 + I3_{trms}^2} \tag{17d}$$

Collective modulation reactive power

The magnitude of the modulation reactive power of a three-phase system depends firstly on the modulation reactive powers of the individual phases, but cannot be determined directly from the values calculated for the single-phase systems.

To illustrate this characteristic, a three-phase system with the current curves shown in Table 5 is considered. Each phase has a modulated current curve (1 A/0 A) with a period of 1 s and a duty cycle of 0.334.

The modulation reactive power per phase amounts to $Q_{mod} = 110\ var$ and neither displacement nor distortion reactive power occurs.

Table 5: Modulation reactive power (three-phase system)

Phase	I	P	T_{on}	Q_{mod}
1	1 A	76 W	0,334 s	110 VAr
2	1 A	76 W	0,334 s	110 VAr
3	1 A	76 W	0,334 s	110 VAr

The modulation reactive power of the three-phase system $Q_{\Sigma,mod}$ depends mainly on how the modulation packets of the individual phases are distributed over time.

The example shown in Figure 5 shows a symmetrically loaded system. The total power P_{Σ} of the three-phase system varies between 0 W and 690 W. In this case, a collective modulation reactive power of $Q_{\Sigma,mod} = 330\ var$ occurs, which equals the sum of the modulation reactive powers of the individual phases.

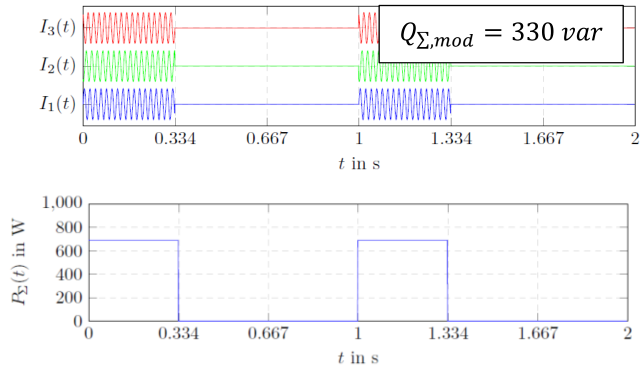

Figure 5: Symmetrical load

If, on the other hand, the oscillation bundles of the individual phases are shifted in relation to each other by a third of the modulation period, as shown in Figure 6, the collective active power P_{Σ} remains constant. Resulting from this, the collective modulation reactive power of the three-phase system is reduced to the value 0, although the individual phases still have a modulation reactive power of 110 Var.

This results in the following relationship for the modulation reactive power:

$$0 \leq Q_{\Sigma,mod} \leq \sum_i Q_{i,mod} \tag{18}$$

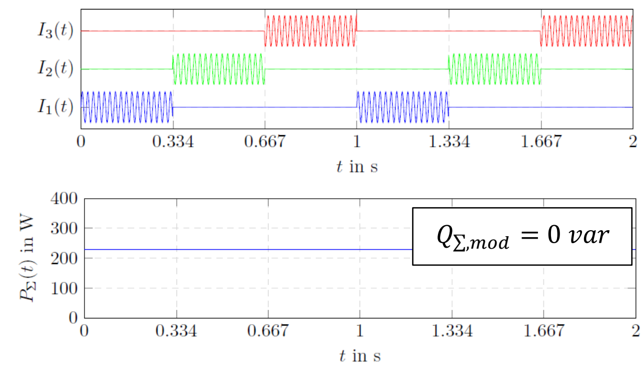

Figure 6: Unsymmetrical load

Unbalance reactive power

In three-phase systems, a total power factor of less than 1 can be achieved, even if each phase is constantly loaded with an ideal load.

An example for this case is shown in Table 6, which describes a 4-wire system with symmetrical voltages and a three-phase ohmic load. The first phase is loaded with 5 A, the second phase with 3 A and phase 3 with 1 A. Under this conditions, the N-wire current amounts to $IN_{tmrs} = 3{,}464\ A$.

Table 6: Example for unbalance reactive power

Phase	U	I	P	S	λ
L1	230 V	5 A	1150 W	1150 VA	1
L2	230 V	3 A	690 W	690 VA	1
L3	230 V	1 A	230 W	230 VA	1
Σ	398,4 V	6,85 A	2070 W	2730 VA	0,76

To determine the reactive power, first the collective apparent power S_Σ and the collective active power P_Σ are calculated.

$$U_{\Sigma\,4L} = \sqrt{230V^2 + 230V^2 + 230V^2} = 398{,}37\text{ V} \quad (19a)$$

$$I_{\Sigma\,4L} = \sqrt{5A^2 + 3A^2 + 1A^2 + 3{,}464A^2} = 6{,}85A \quad (19b)$$

$$S_\Sigma = 398{,}37\text{ V} \cdot 6{,}85\,A = 2730\,VA \quad (19c)$$

$$P_\Sigma = 1150\text{ W} + 690\,W + 230\,W = 2070\,W \quad (19d)$$

Then the reactive power can be calculated according to equation 7 resulting in a power factor of $\lambda = 0{,}76$.

$$Q = \sqrt{2730\,VA^2 - 2070\,W^2} = 1780\,VAr \quad (19e)$$

The effects of the asymmetrical load on the three-phase power transmission become clear when comparing the total power with a symmetrical system (Abbildung 8).

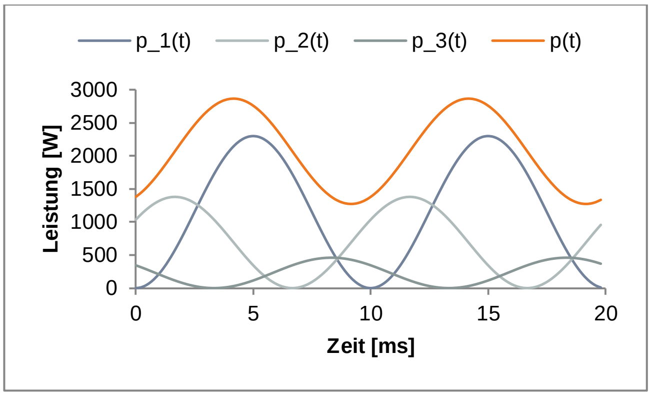

Figure 7: Instantaneous power in three-wire system (unbalanced)

In the unbalanced case (see Figure 7), the instantaneous power $p(t)$ varies between 1450 W and 2800 W within a net frequency period. The line between source and load is therefore not optimally utilized over time, since the current power temporarily exceeds or falls short of the average power of 2070 W.

In the symmetrical case, which is shown in Abbildung 8 for the same average power, the instantaneous total power $p(t)$ lies constant at 2070 W. This makes ideal use of the transmission line and the power factor is 1.

The collective current I_Σ reduces in this case to 5,196 A.

$$I_{\Sigma\,4L} = \sqrt{3A^2 + 3A^2 + 3A^2 + 0A^2} = 5{,}196A \quad (20)$$

Phase	U	I	P	S	λ
L1	230 V	3 A	690 W	690 VA	1
L2	230 V	3 A	690 W	690 VA	1
L3	230 V	3 A	690 W	690 VA	1
Σ	398,4 V	5,196 A	2070 W	2070 VA	1

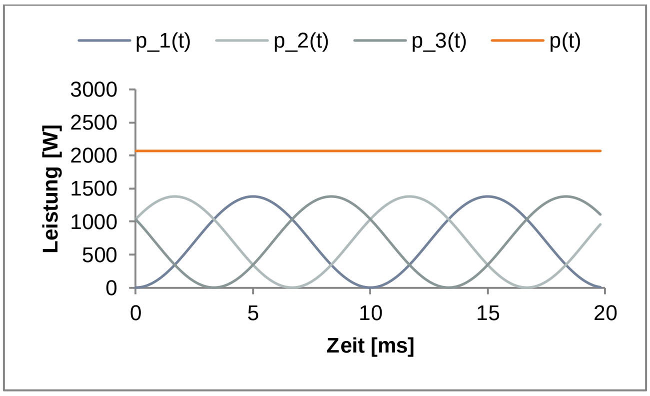

Abbildung 8: Momentanleistung im Dreiphasensystem (symmetrisch)

Power measurement in practical use

The mobile power quality analyzers from A-Eberle (*PQ-Box 100, 150, 200* and *300*) offer a variety of setting options to correctly measure the different power types depending on the specific use case. These options are shown in Figure 9.

For the recording of long-term data and the corresponding calculation of apparent, reactive and active powers there are two freely parameterizable measurement intervals. The N-second interval can be set in a range from 1 s to 600 s. In addition, power and energy quantities are recorded at standard intervals of 10, 15 or 30 minutes.

To determine the cyclic measurement data in the selected data classes, all described reactive power types (displacement, distortion, modulation and unbalance) are computed at all time.

When calculating the collective reactive and apparent power, it is also possible to select, whether modulation and/or unbalance reactive power should be taken into account (depending on the application).

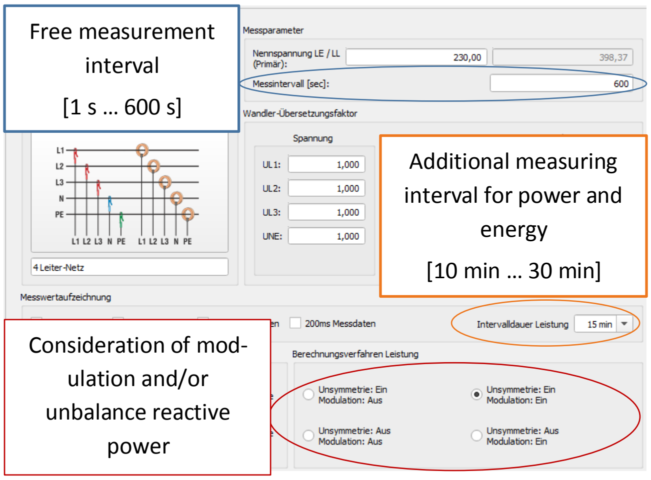

Figure 9: Basic settings PQ-Box

Selection of calculation methods according to the application

While displacement and distortion reactive power always have an informative value at each measuring point, the boundary conditions of the measurement and the position of the measuring point in the network must be taken into account in the measurement of unbalance reactive power in order to obtain meaningful measurement data.

As an example, Figure 10 shows a 400 V suburban grid including a 20 kV/400 V transformer, mainly single-phase connected households and an industrial plant with two-phase load (e.g. welding unit)

Thereby, the most important factor for measuring unbalance is the length of the cable section, which is loaded by the measured reactive power.

When measuring in a housing area where the individual loads are mainly connected in a single-phase manner, very large local asymmetries may occur. However, the measurable unbalance reactive power has no great significance here, since the load of the three-phase system changes at each nearby node and therefore no larger network section is loaded asymmetrically.

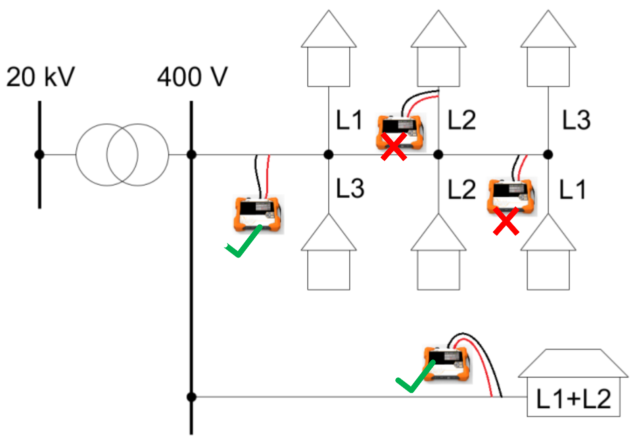

Figure 10: Reasonable measurement of unbalance reactive power

However, it makes sense to consider unbalance reactive power for measurements directly at the local grid transformer or at long supply lines for unbalanced loads, as shown in Figure 10.

The measurement of modulation reactive power depends strongly on the selected measurement interval. Taking this reactive power into account for the calculation of the collective apparent power is therefore only useful if loads with periodically fluctuating power are present and the modulation frequency of the power fluctuation is known.

Interpretation of measurement results

In the *WinPQmobil* analysis and evaluation software, the power components are grouped according to their type. The fundamental reactive power in Figure 11 displays the fundamental displacement reactive power *QV* of each phase. The sign of this reactive power represents its type, where negative values represent capacitive loads and positive values represent inductive loads.

Furthermore, the sum of the reactive power of the single phases always returns the total reactive displacement power Q total.

The distortion reactive power D has no sign and can also be summed up by simply adding the single phase quantities (siehe Figure 11).

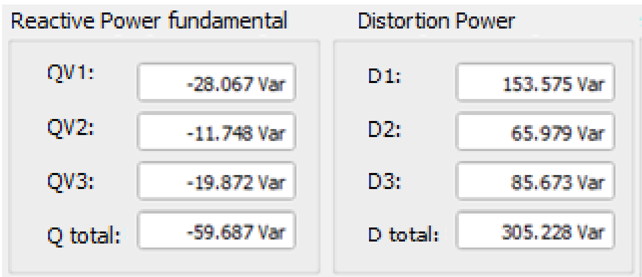

Figure 11: Fundamental and distortion reactive power

The situation is different with modulation reactive power. As described in the section "collective modulation reactive power", the total reactive power Q mod total cannot be determined directly from the single phase quantities, but depends on the time curve of the phase powers relative to each other.

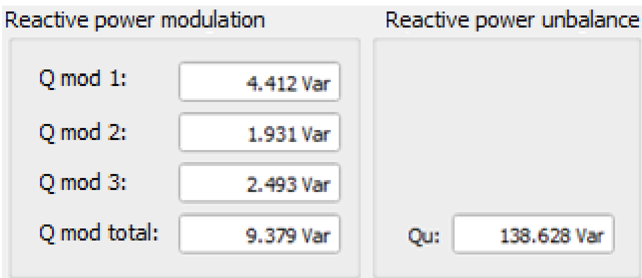

Figure 12: Modulation and unbalance reactive power

The collective reactive power is calculated according to the relationship of equation 8, taking into account only the quantities defined in the basic settings of the measuring device (see Table 7).

Table 7: Reactive power calculation depending on the selected calculation method

Setting	Calculation method
No unbalance No modulation	$Q_\Sigma = \sqrt{{Q_{h1}}^2 + Q_d^2}$
Unbalance No modulation	$Q_\Sigma = \sqrt{{Q_{h1}}^2 + Q_d^2 + Q_u^2}$
No unbalance Modulation	$Q_\Sigma = \sqrt{{Q_{h1}}^2 + Q_d^2 + Q_m^2}$
Unbalance Modulation	$Q_\Sigma = \sqrt{{Q_{h1}}^2 + Q_d^2 + Q_m^2 + Q_u^2}$

This power is then used to calculate the apparent power per phase and the apparent collective power of the three-phase system, as shown in Figure 13.

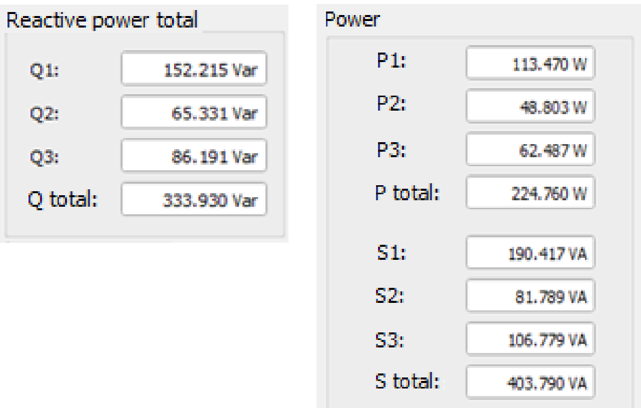

Figure 13: Collective active, reactive and apparent power

Table 8: Overview of all reactive power components in single-phase and three-phase system

Quantity	Single-phase system	Three-phase system
Fundamental displacement reactive power	x	x[2]
Displacement reactive power harmonics	x	x[3]
Distortion reactive power	x	x[3]
Modulation reactive power	x	x
Unbalance reactive power	-	x

[2] Indirect measurement: Calculation by adding up the single phase quantities